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13. ABSTRACT (Maximum 200 words)

The major problem for the present contract period was to extend fast algorithms based on displacement structure to matrices with zero minors - the socalled singular case. Almost all the literature, of about a hundred years, deals with the socalled regular or nonsingular cases, with particular success in the case of Hankel and Hankel-related matrices. These results are related to the now wellknown Berlekamp-Massey algorithm (for solving Hankel linear equations).

For Toeplitz and Toeplitz-related matrices, there were only some partial and rather complicated solutions. In the Ph.D. research of D. Pal a complete and elegant solution is given to this problem for the case of Toeplitz and quasi-Toeplitz matrices. While not as general as one would have liked, the latter class of matrices allowed one to get the first general solution to the much-studied stability and root-distribution problems for discrete-time systems. Additional results appear in the list of publications in the appendix.

14. SUBJECT TERMS

Estimation Theory, Fast Algorithms, Hankel Matrices, Toeplitz Matrices, Displacement Structures

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Studies in Estimation Theory, Applications and Implementation

Final Report

by

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May 15, 1992

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This is the final report on ARO Contract DAAL-89-K-0109, May 16, 1989 - May 15, 1992.

A substantial amount of work was accomplished during the contract period, as may be seen from the list of publications in the Appendix. Of course, because of the various delays and lags in the publications process, several of the papers relate to work initiated and or completed in earlier contract periods.

The major problem posed in our research proposal for the present contract period was to extend fast algorithms based on displacement structure to matrices with zero minors – the socalled singular case. Almost all the literature, of about a hundred years, deals with the socalled regular or nonsingular cases, with particular success in the case of Hankel and Hankel-related matrices. These results are related to the now wellknown Berlekamp-Massey algorithm (for solving Hankel linear equations).

For Toeplitz and Toeplitz-related matrices, there were only some partial and rather complicated solutions. In the Ph.D. research of D. Pal we have given a complete and elegant solution to this problem for the case of Toeplitz and quasi-Toeplitz matrices. While not as general as we would have liked, the latter class of matrices allowed us to get the first general solution to the much-studied stability and root-distribution problems for discrete-time systems. A paper on this last result has been accepted by the *IEEE Transactions on Automatic Control*, while a paper on the underlying mathematics has been accepted by the *SIAM Journal on Matrix Analysis.* These researches spurred related studies on stability and root distribution problems, with a long paper giving a unified approach to half-plane (or continuous-time system) and unit disc (discrete-time system) problems appearing in the February 1991 issue of the *IEEE Transactions on Circuits and Systems.* In that paper, we had studied only the regular (or nonsingular) case, but were able to show that specializing our earlier general fast algorithms for matrices with displacement structure allowed us to get a natural approach unifying the Classical Routh-Hurwitz half-plane tests and the classical Schur-Cohn unit disc tests. Furthermore it showed that these were only two of a whole family of different-looking, but computationally equivalent tests. We also obtained a framework for classifying all possible tests, thus for example including some recent tests of Lepschy et al. (1988). In Pal's thesis we used our extension of the displacement structure theory to handle the singular cases of the stability/root distribution problem.

Apart from the above major effort on the main problem put forward in our proposal, we continued to build upon work in earlier periods, especially extending the techniques of Chun (Ph.D. thesis, June 1989) to apply to other classes of problems. In particular, Pal used these ideas to find new ways of solving linear equations with singular Toeplitz and quasi-Toeplitz matrices. This work has been submitted for publication. Recently with another student, A. Sayed, the ideas have been applied to the currently much studied QR adaptive filtering algorithm. We have thus obtained a new computational array for updating the weight vector in the adaptive algorithm, impoving on the much-cited McWhirter array (see, *e.g.* the textbook of S. Haykin, Adaptive Filter Theory, 2nd edition, Prentice-Hall 1991).

In a related effort, we made some connections with the Ph.D. studies of M. Genossar on the time-frequency analysis of nonstationary processes. A particularly important subclass of such processes are the cyclostationary processes, whose covariance matrices have a particular kind of block-Toeplitz structure. In the course of studying such processes, we made a complete analysis of the statistical estimation procedure for the socalled cyclic autocorrelation (or autocovariance) matrix of such processes. We have extended the earlier work of Hurd and Gardner on this problem; a paper on our results is to appear in the *IEEE Transactions on Information Theory.*

Our other major effort has been on fast algorithms for subspace computation in the new high resolution sensor-array processing algorithms such as MUSIC, ESPRIT and WSF. In the thesis work of G. Xu a new socalled FSD (Fast subspace decomposition) algorithm was developed for this purpose. This led us to see how the ideas of displacement structure could be used to further speed up the FSD computations. Some papers on these results – and their connections to certain classes of eigenvalue and singular value problems – are under review.

Finally we may mention that during the present contract period, we worked on revising and resubmitting papers submitted in earlier periods. The publications list shows that this has been a major activity. Among several interesting papers, we may make mention of some papers coauthored by a previous postdoctoral scholar, A. Dembo (see [5],[6],[13],[17] in the list of publications) and work on regular iterative arrays for VLSI design (see [4],[9],[12],[21]).

The Ph.D. dissertations completed in this period were those of D. Pal (May 1990), G. Xu (Sept. 1991), and M. Genossar (April 1992); abstracts are appended to this report.

Scientific Personnel Supported by this Project and Degrees Awarded during 1989-1992:

Professor T. Kailath
Graduate Students: M. Genossar, J. Gisladottir, B. Khalaj, D. Pal,
A. Sayed, G. Xu
Research Associates: H. Lev-Ari, T. Constantinescu, G. Xu

Advanced Degrees

[1] D. Pal, *Fast Algorithms for Structured Matrices With Arbitrary Rank Profile*, Ph.D. Dissertation, Stanford University, May 1990.

[2] G. Xu, *Fast Subspace Decomposition and Its Applications*, Ph.D. Dissertation, Stanford University, September 1991.

[3] M. Genossar *Spectral Characterizations of Nonstationary Processes*, Ph.D. Dissertation, Stanford University, April 1992.

Appendix

ARO Supported Publications

Proposal Number: DAAL03-89-K-0109
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Published Journal Papers

[1] Y Bistritz, H. Lev-Ari, and T. Kailath, Immittance-Domain Levinson Algorithms, *IEEE Trans. Info. Thy.*, **35**(3):675–682, May 1989.

[2] J. Chun and T. Kailath, A Constructive Proof of the Gohberg-Semencul Formula, *Linear Algebra Appl.*, **121**:475–489, August 1989.

[3] Y. Bistritz, H. Lev-Ari, and T. Kailath, Immittance Versus Scattering Domains Fast Algorithms for Non-Hermitian Toeplitz and Quasi-Toeplitz Matrices, *Linear Algebra Appl.*, **122/123/124**:847–888, September 1989.

[4] V.P. Roychowdhury and T. Kailath, Subspace Scheduling and Parallel Implementation of Non-Systolic Regular Iterative Algorithms, *J. VLSI Signal Process.*, **I**(127-142), October 1989.

[5] A. Dembo, Simple Proof of the Concavity of the Entropy Power with Respect to Added Gaussian Noise, *IEEE Trans. Info. Thy.*, **35**(4):887–888, July 1989.

[6] A. Dembo, On Gaussian Feedback Capacity, *IEEE Trans. Info. Thy.*, **35**(2):1072–1076, September 1989.

[7] J. Gisladottir, H. Lev-Ari, and T. Kailath, Orthogonal Realization of First-Order Allpass Filters for Two-Dimensional Signals, *Multidimensional Systems and Signal Process.*, **1**(1):39–50, March 1990.

[8] V. P. Roychowdhury, J. Bruck, and T. Kailath, Efficient Algorithms for Reconfiguration in VLSI/WSI Arrays, *IEEE Trans. Comput.*, **39**(4):480–489, April 1990.

[9] J. M. Jover, T. Kailath, H. Lev-Ari, and S. K. Rao, On the Analysis of Synchronous Computing Systems, *SIAM J. Comput.*, **19**(4):627–643, August 1990.

[10] H. Lev-Ari, volume 22 of *Institute for Math. and Its Appls.* chapter Extension Problems Under the Displacement Structure Regime, pages 211–227, Springer-Verlag, New York, Signal Processing, Part I: Signal Processing Theory Edition, 1990.

[11] J. Chun and T. Kailath, volume 22 of *Institute for Math and Its Appls.*, chapter Displacement Structure for Hankel, Vandermonde and Related (Derived) Matrices, pages 37–58, Springer-Verlag, New York, Signal Processing, Part I: Signal Processing Theory Edition, 1990.

[12] J. Chun T. Kailath and V. Roychowdhury, *Systolic Array for Solving Toeplitz Systems of Equations*, pages 547–560, Oxford & IBH Publishing Co., India, Spectral Analysis in One or Two Dimensions, eds. S. Prasad and R. Kashyap, 1990.

[13] A. Dembo and O. Zeitouni, Maximum A Posteriori Estimation of Elliptic Gaussian Fields Observed Via a Noisy Nonlinear Channel, *J. Multivariate Anal.*, **35**(2):151-167, Nov 1990.

[14] J. Chun and T. Kailath, Vol. F70 of *NATO ASI Series*, Generalized Displacement Structure for Block-Toeplitz, Toeplitz-block, and Toeplitz-derived Matrices, pages 215–236, Springer-Verlag, Berlin, Numerical Linear Algebra, Digital Signal Processing and Parallel Algorithms, 1991.

[15] J. Chun and T. Kailath, Divide-and-Conquer Solutions of Least-Squares Problems for Matrices with Displacement Structure, *SIAM J. Matrix Anal. and Appl.*, **12**(1):128–145, January 1991.

[16] H. Lev-Ari, Y. Bistritz, and T. Kailath, Generalized (Bezoutians) and Families of Efficient Root-Location Procedures, *IEEE Trans. Circuits and Systems*, **38**(2):170–186, February 1991.

[17] A. Dembo and O. Zeitouni, Onsager-Machlup Functional and Maximum A Posteriori Estimation for A Class of Non-Gaussian Random Fields, *J. Multivariate Anal.*, **36**(2):243–262, February 1991.

[18] J. Chun and T. Kailath, Displacement Structure for Hankel, Vandermonde and Related (Derived) Matrices, *Linear Algebra Appl.*, **151**:199–227, 1991.

[19] J. Chun and T. Kailath, Systolic Array Implementation of the Square Root Chandrasekhar Filter, *Internat. J. Control*, **54**(6):1385–1398, December 1991.

[20] Y. Bistritz, H. Lev-Ari, and T. Kailath, Immittance-Domain Three-Term Schur and Levinson Recursions for Quasi-Toeplitz Complex Hermitian Matrices, *SIAM J. Matrix Anal. and Appl.*, **12**(3):497–520, July 1991.

[21] H. V. Jagadish and T. Kailath, Obtaining Schedules for Digital Systems, *IEEE Trans. Acoust. Speech Signal Process.*, **39**(10):2296–2316, October 1991.

[22] T. Kailath, *From Kalman Filtering to Innovations, Martingales, Scattering and Other Nice Things*, pages 55–88, Mathematical System Theory: The Influence of R.E. Kalman, ed. A.C. Antoulas. Springer-Verlag, Communications and Control Engineering Edition, 1991.

[23] T. Kailath Remarks on the Origin of the Displacement Rank Concept, *J. Appl. Math. Computation*, **45**(2):193–206, September 1991.

[24] H. Lev-Ari and T. Kailath, State-Space Approach to Factorization of Lossless Transfer Functions and Structured Matrices, *Linear Algebra Appl.*, **162–164**:273–295, February 1992.

[25] R. Ackner, H. Lev-Ari and T. Kailath, Transmission-Line Models for the Modified Schur Algorithm, *IEEE Trans. on Circuits and Systems*, **39**(4):290–296, April 1992.

Papers Accepted for Publication

[26] A. Bruckstein, T. Kailath, I. Koltracht, and P. Lancaster, On The Reconstruction of Layered Media From Reflection Data, *SIAM J. Matrix Anal. and Appl.*

[27] R. Ackner and T. Kailath, On The Ptak-Young Generalization of the Schur-Cohn Theorem, *IEEE Trans. Automat. Control.*

[28] D. Pal and T. Kailath, A Fast-Matrix-Factorization Approach Towards the Singular Cases of Tabular Form Root Distribution Procedures: The Unit Circle Case, *IEEE Trans. Automat. Control*, 1992.

[29] T. Kailath and J. Chun, Generalized Displacement Structure for Block-Toeplitz, Toeplitz-Block, and Toeplitz-Derived Matrices, *SIAM J. Matrix Anal. and Appl.*

[30] R. Ackner, H. Lev-Ari, and T. Kailath, The Schur Algorithm for Matrix-Valued Meromorphic Functions, *SIAM J. Matrix Anal. and Appl.*

[31] D. Pal, Gohberg-semencul type formulas via embedding of lyapunov equations, *IEEE Trans. Acoust. Speech Signal Process.*

[32] M. Genossar, H. Lev-Ari, and T. Kailath, Consistent Estimation of Cyclic Autocorrelation, *IEEE Trans. Acoust. Speech Signal Process.*

[33] V. P. Roychowdhury, L. Thiele, S. K.Rao, and T. Kailath, On the Localization of Algorithms for VLSI Processor Arrays, *IEEE Trans. Comput.*

Papers Under Review

[34] G. Xu and T. Kailath, Fast Algorithms for Computing the Principal Singular Values and Vectors, *SIAM J. Matrix Anal. and Appl.*

[35] G. Xu and T. Kailath, Fast Decomposition of the Principal Eigenspace Via Exploitation of Eigenvalue Multiplicity, *SIAM J. Matrix Anal. and Appl.*

[36] D. Pal and T. Kailath, Fast Triangular Factorization and Inversion of Hankel and Related Matrices With Arbitrary Rank Profile, *SIAM J. Matrix Anal. and Appl.*

[37] V. P. Roychowdhury and T. Kailath, Regular Processor Arrays for Matrix Algorithms with Pivoting, *ACM Communications.*

[38] Guanghan Xu and Thomas Kailath, Fast Signal-Subspace Decomposition, Pt. I: Ideal Covariance Matrices, *IEEE Trans. Acoust. Speech Signal Process.*

[39] G. Xu and T. Kailath, Fast Signal-Subspace Decomposition, Pt. II: Sample Covariance Matrices, *IEEE Trans. Acoust. Speech Signal Process.*

[40] D. Pal and T. Kailath, A Fast Matrix Factorization Approach Towards the Singular Cases of Tabular Form Root Distribution Procedures: The Imaginary Axis Case, *IEEE Trans. Circuits and Systems*, 1991.

[41] G. Xu, S. D. Silverstein, R. H. Roy, and T. Kailath, Beamspace ESPRIT, *IEEE Trans. Acoust. Speech Signal Process.*

[42] A. Dembo, A New Proof of the Entropy Power Inequality, *IEEE Trans. on Information Theory*

[43] M. Goldburg, M. Genossar, and T. Kailath, Optimal Coefficient Selection for Discrete Time Wavelet Filters, *IEEE Trans. on Signal Processing*

[44] G. Xu, R.H. Roy, and T. Kailath, Detection of Number of Sources Via Exploitation of Centro-Symmetry Property, *IEEE Trans. on Signal Processing*

Conference Papers

[45] T. Kailath and V.P. Roychowdhury, Scheduling linearly indexed assignment code,, pages 118–129, Los Angeles, CA, January 1989. Proc. SPIE.

[46] D. Pal and T. Kailath, Fast modified triangular factorization of Hankel, Quasi-Hankel and Sign-Modified Quasi-Hankel matrices with arbitrary rank profile, pages 614–618, Asilomar, CA, October 1989. 23rd Asilomar Conference on Circuits, Signals and Computers.

[47] T. Varvarigou, V. P. Roychowdhury, and T. Kailath, Some new algorithms for reconfiguring VLSI/WSI arrays, pages 229–235, San Francisco, CA, Jan 1990. Intern'l. Conference on Wafer Scale Integration.

[48] D. Pal and T. Kailath, A generalization of the Schur-Cohn test: The singular case, Canada, August 1990. 33rd Midwest Symposium on Circuits and Systems.

[49] D. Pal and T. Kailath, Approximate AR modeling: An alternative to regularization via Schur algorithm for matrices with arbitrary rank profile, pages 275–279, New York, October 1990. 5th Acoustics, Speech and Signal Processing Workshop on Spectrum Estimation and Modeling.

[50] G. Xu and T. Kailath, Fast signal subspace decomposition without eigendecomposition, Asilomar, CA, November 1990. 24th Asilomar Conference on Circuits, Signals and Computers, 966-969.

[51] G. Xu and T. Kailath, A simple and effective algorithm for estimating time delay of communications signals, Hawaii, November 1990. Int'l. Symp. on Info. Thy. and Its Appls., 267-270.

[52] D. Pal and T. Kailath, Approximate ar modeling: A schur approach, Rochester, NY, October 1990. 5th ASSP Workshop on Spectrum Estimation and Modeling.

[53] G. Xu and T. Kailath, A Fast Algorithm for Signal Subspace Decomposition and Its Performance Analysis, pages 3069–3072, Toronto, Canada, May 1991. 1991 Intern'l. Conf. Acoustics, Speech and Signal Processing.

[54] G. Xu, S. Silverstein, R. Roy, and T. Kailath, Parallel implementation and performance analysis of beamspace esprit, Toronto, Canada, May, 1991 1991. ICASSP-91.

[55] T. Kailath, Root distribution and displacement structure, Chicago, IL, June 1992. Symposium on Fundamentals of Discrete-Time Systems.

[56] Debajyoti Pal and Thomas Kailath, A systematic generalization of the schur-cohn procedure for root localization with respect to the unit circle: The singular cases, Chicago, IL, June 1992. Symposium on Fundamentals of discrete-Time Systems.

[57] G. Xu, R. H. Roy, and T. Kailath, Detection of source number with centro-symmetric arrays, 1992 Intern'l. Conf. Acoustics, Speech and Signal Processing, 1992.

Theses

[58] D. Pal, *Fast Algorithms for Structured Matrices With Arbitrary Rank Profile*, Ph.D. Dissertation, Stanford University, May 1990.

[59] G. Xu, *Fast Subspace Decomposition and Its Applications*, Ph.D. Dissertation, Stanford University, September 1991.

[60] M. Genossar *Spectral Characterizations of Nonstationary Processes*, Ph.D. Dissertation, Stanford University, April 1992.

D. Pal, Fast Algorithms for Structured Matrices with Arbitrary Rank Profile, Ph.D. Dissertation, Stanford University, May 1990.

Abstract

Triangular factorization, solution to linear equations, inversion, computation of rank profile and inertia (in the Hermitian case) etc. of general $n \times n$ matrices require $O(n^3)$ operations. For certain structured matrices including Toeplitz and Hankel matrices the computational complexity is known to be $O(n^2)$ or better. These structured matrices often arise in a wide variety of areas including Signal processing, Systems theory and Communications. Fast (i.e. $O(n^2)$) algorithms for these structured matrices have been actively studied for over twenty five years. However almost all the authors have assumed that the underlying matrices are strongly regular i.e. every principal submatrix is nonsingular. Although some fast algorithms have recently been developed for certain problems involving some of these structured matrices which may have one or more zero minors, several other problems remain unresolved; also a unified approach towards these problems is lacking. In this dissertation, we obtain several new results through a unified approach to the problems mentioned earlier.

First we derive fast (i.e. $O(n^2)$) procedures for computing a "modified" triangular factorization which is a LDU factorization where L is lower triangular (resp. U is upper triangular) with unit diagonal entries and D is a block diagonal matrix with possibly varying block sizes. Only strongly nonsingular matrices will always have a purely diagonal nonsingular D matrix. For the matrices we study the diagonal blocks are also structured: in particular they are Hankel (resp. Quasi-Toeplitz) for Hankel (resp. Toeplitz) and Quasi-Hankel (resp. Quasi-Toeplitz) matrices.

A particular application of our result is a fast method of computing rank profile and inertia, leading to alternative proofs of certain results due to Iohvidov (1974) on rank profile

and inertia of Hankel and Toeplitz matrices.

Next using the results on modified triangular factorization we extended the Schur complement based approach of Chun [Chu89] for inversion of strongly regular Toeplitz and Hankel matrices to Hermitian Toeplitz, Quasi-Toeplitz, Hankel, and Quasi-Hankel matrices with arbitrary rank profile.

We apply our procedures for computing inertia to derive $O(n^2)$ procedures for computing root distribution of an n-th order polynomial with respect to the unit circle and the imaginary axis. In particular it has been possible to derive the first general recursive procedure for determining the root-distribution of a polynomial with respect to the unit circle: the classical Schur-Cohn test for this problem fails in the presence of certain "singularities," which correspond to a Quasi-Toeplitz Bezoutian matrix being non-strongly-regular.

G. Xu, Fast Subspace Decomposition and its Applications, Ph.D. Dissertation, Stanford University, September 1991.

Abstract

IN THE AREA OF DETECTION AND ESTIMATION, there is a class of so-called *signal subspace* algorithms that has various applications in mobile/cellular communications, target tracking and signal estimation, system identification, ARMA modeling, adaptive filtering. Though signal subspace algorithms have been shown to perform significantly better than the traditional least-squares methods, most of them have been used *only* for *off-line* analyses instead of being realized *on-line*. The main reason is that they all incorporate the common key step of estimating the signal subspace, called *signal subspace decomposition*, which is conventionally achieved by computationally intensive eigendecomposition (ED) or singular value decomposition (SVD). The ED or SVD of an $M \times M$ matrix requires at least $O(M^3)$ multiplications which is quite significant for large M. Also in many of its more stable instantiations, the ED (or SVD) involve global communication and significant amounts of local storage, properties that make VLSI parallel implementation difficult. Therefore, the ED (or SVD) represents a significant barrier that prevents on-line realizations. Orders of magnitude computational reduction of signal subspace decomposition may lead to *real-time* application of various signal subspace algorithms and thus result in significant progress in the fields of signal processing and communications, among others.

In this thesis, we present a class of Fast Subspace Decomposition (FSD) algorithms by exploiting the matrix structure associated with signal subspace algorithms. The new FSD techniques, based on the well-known Lanczos algorithm for tridiagonalization, require only $O(M^2 d)$ multiplications for an $M \times M$ matrix, where d is the dimension of the signal subspace. In the aforementioned applications, M is usually

much larger than d and FSD achieves an order of magnitude reduction in computational complexity. If the matrix under consideration has additional structure common in signal processing and communications, *e.g.,* Toeplitz and Hankel, FSD can exploit such structures and achieve another order computational reduction. New detection schemes for estimating d are also presented and can be carried out in the process of signal subspace decomposition. More importantly, the FSD approach can be easily implemented in parallel using simple array processors, and the computation time can be reduced further to $O(Md)$ or $O(\log Md)$ using $O(M)$ or $O(M^2)$ multipliers, respectively.

This thesis also presents some new results in numerical linear algebra, based on which the FSD is derived. In particular, a new error estimate of the Rayleigh-Ritz approximation, a key step of the Lanczos algorithm, is derived; this error estimate is much tighter than the well-known Kaniel and Saad bounds for the matrix of interest. Combining these new results with the theory of multivariate statistical analysis, enables a rigorous performance analysis of the FSD approach. Unlike many fast algorithms that trade performance for speed, the performance analysis shows that FSD has the ***same asymptotic*** performance as its more costly counterparts, ED and SVD, and that the new detection schemes are ***strongly consistent.***

M. Genossar, Spectral Characterizations of Nonstationary Processes,
Ph.D. Dissertation, Stanford University, April 1992

Abstract

Nonstationary random processes are encountered in many fields. Various approaches have been suggested for defining spectra for nonstationary processes. Among them are the short time Fourier transform, the Wigner-Ville distribution, the asymptotic spectrum, the cyclic autocorrelation, the cyclic spectrum and Loève's harmonizable representation.

This dissertation is concerned with modeling, characterizing and estimating spectra for nonstationary discrete-time processes, in a probabilistic framework. Our focus is on persistent processes, and within those we are particularly interested in processes that contain some periodic structure.

First we develop the probabilistic theory for nonstationary processes in the harmonizable framework, which is based on a bivariate Fourier transform. We discuss the properties of various nonstationary processes in this framework, and show how other spectral representations of nonstationary processes (the Wigner-Ville distribution, the cyclic autocorrelation and the cyclic spectrum) fit into this framework. We classify processes according to their spectral support, and discuss decomposition of the bivariate spectral measure.

Next we present a statistical analysis of an estimator of the cyclic autocorrelation for nonstationary Gaussian processes. We derive new necessary and sufficient conditions for consistency in mean square of the estimator, and discuss rate of convergence of the estimator.

Finally we consider models and spectral relations for cyclostationary processes. Gladyshev has suggested a correspondence between cyclostationary processes and

multichannel stationary processes. For this correspondence we derive relations between the spectral representations of the cyclostationary process and of the multichannel stationary process. This provides ties between the theory of multichannel stationary processes and the theory of cyclostationary processes. Periodic autoregressive and periodic autoregressive moving-average models are sometimes used to model cyclostationary processes. We derive relations between the parameters of such models and the cyclic spectrum of the cyclostationary process. These relations can be used for parametric estimation of the spectrum of cyclostationary processes.